Mathematics: analysis and approaches
Standard Level
Paper 2

Name

Date: _____

1 hour 30 minutes

_____

Instructions to candidates

- Write your name in the box above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all of Section A in the spaces provided.
- Section B: answer all of Section B on the answer sheets provided. Write your name on each answer sheet and attach them to this examination paper.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: analysis and approaches formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[80 marks]**.

exam: 11 pages

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions in the boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 6]

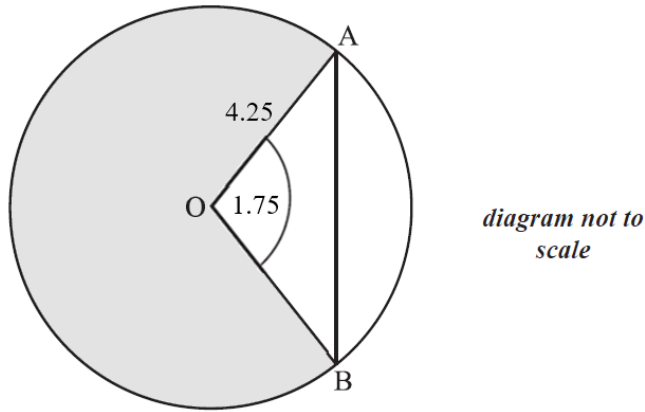In an arithmetic sequence, $S_{30} = 1560$ and $u_{30} = 110$. Find the value of u_1 and the value of d.

2. [Maximum mark: 7]

The circle shown below has center O and radius measuring 4.25 cm.

Points A and B lie on the circle and angle AOB measures 1.75 radians.

(a) Find AB.   [3]

(b) Find the area of the shaded region.   [4]

3. [Maximum mark: 6]

A multiple-choice test consists of 12 questions. Each question has four answers from which to choose. Only one of the answers is correct. For each question, Boris randomly chooses one of the four answers.

(a) Write down the expected number of questions Boris answers correctly.   [1]

(b) Find the probability that Boris answers exactly three questions correctly.   [2]

(c) Find the probability that Boris answers more than three questions correctly.   [3]

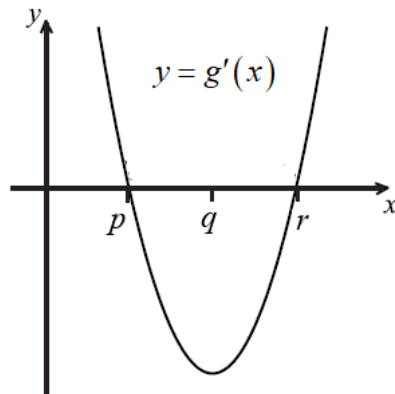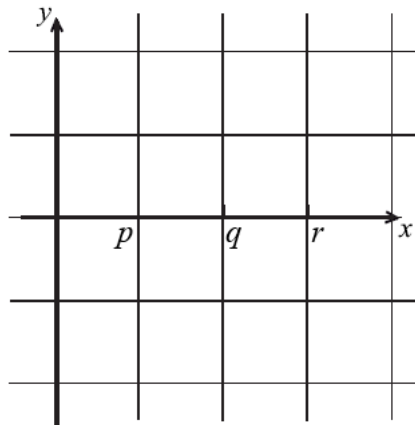**4.** [Maximum mark: 6]

The diagram below shows part of the graph of the **gradient** function, $y = g'(x)$.

(a) On the grid below, sketch a graph of $y = g''(x)$, clearly indicating the *x*-intercept. [2]

(b) Complete the table below, for the graph of $y = g(x)$. [2]

	x-coordinate
(i) maximum point on g	
(ii) minimum point on g	

(c) Justify your answer to part (b) (ii). [2]

5. [Maximum mark: 7]

Given that events A and B are independent, $\mathrm{P}(B) = 2\mathrm{P}(A)$, and $\mathrm{P}(A \cup B) = 0.72$, find $\mathrm{P}(B)$.
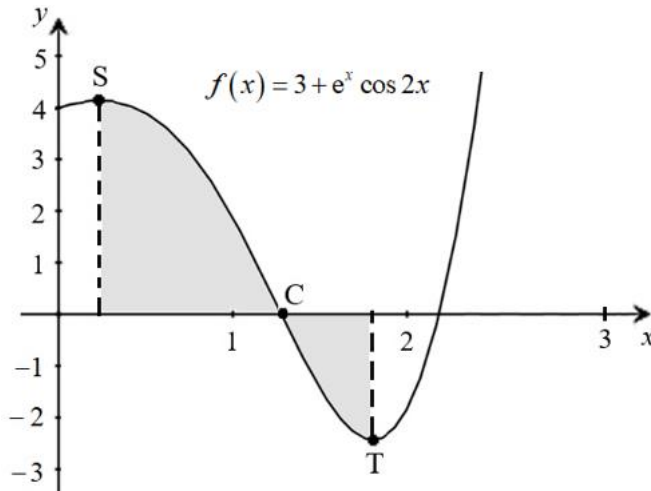
6. [Maximum mark: 6]

Let $f(x) = 3 + e^x \cos 2x$, for $0 \le x \le 3$. A portion of the graph of f is shown below.

There is an x-intercept at the point C, a local maximum point at S where $x = s$, and a minimum point at T where $x = t$.

(a) Write down the following:

    (i)   the x-coordinate of C;

    (ii) the value of s;

    (iii) the value of t.   [3]

(b) (i)   Let $\int_s^t f(x)\,dx = k$. Calculate the value of k.

    (ii) Explain why k is **not** the area of the shaded region.   [3]

Question 6 continues on the next page

Question 6 continued

Do **not** write solutions on this page.

Section B

Answer **all** the questions on the answer sheets provided. Please start each question on a new page.

7. [Maximum mark: 15]

A farmer has an operation growing button mushrooms indoors that are sold at a local market. On a particular day, the farmer harvests 200 button mushrooms and measures the diameter (d) of each mushroom in centimeters. The results are shown in the frequency table below.

diameter, d cm	$0 < d \leq 1$	$1 < d \leq 2$	$2 < d \leq 3$	$3 < d \leq 4$	$4 < d \leq 6$	$6 < d \leq 7.5$	$7.5 < d \leq 10$
frequency	30	40	50	30	33	11	6

(a) Calculate an estimate for the mean of the diameters of the mushrooms.   [3]

(b) A cumulative frequency graph is given below for the diameters of the mushrooms.
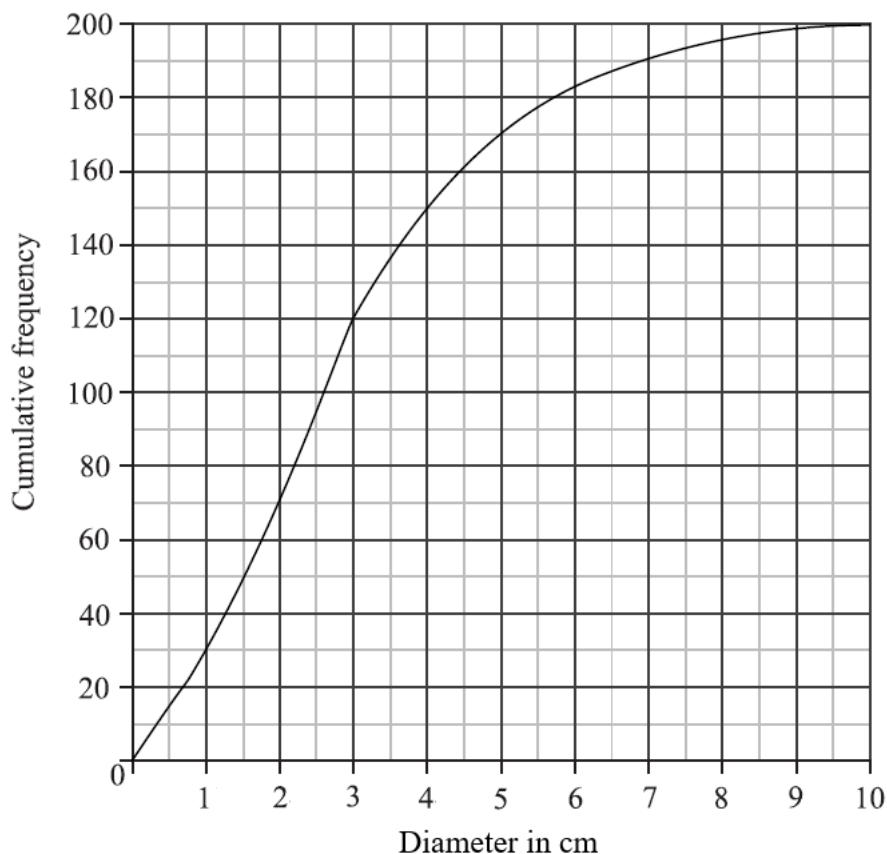
Use the graph to answer the following.

(i) Estimate the interquartile range.

(ii) Given that 20% of the mushrooms have a diameter more than k cm, find the value of k.   [6]

Question 7 continues on the next page

Question 7 continued

In preparation for selling the mushrooms, the farmer classifies each of them as *small*, *medium* or *large* using the following criteria.

Small: diameter is less than 2 cm
Medium: diameter is greater than or equal to 2 cm but less than 6 cm
Large: diameter is greater than or equal to 6 cm

(c) Write down the probability that a mushroom randomly selected from the day's harvest is *Small*. [2]

The cost of a *Small* mushroom is $\$0.10$, a *Medium* mushroom is $\$0.15$ and a *Large* mushroom is $\$0.25$.
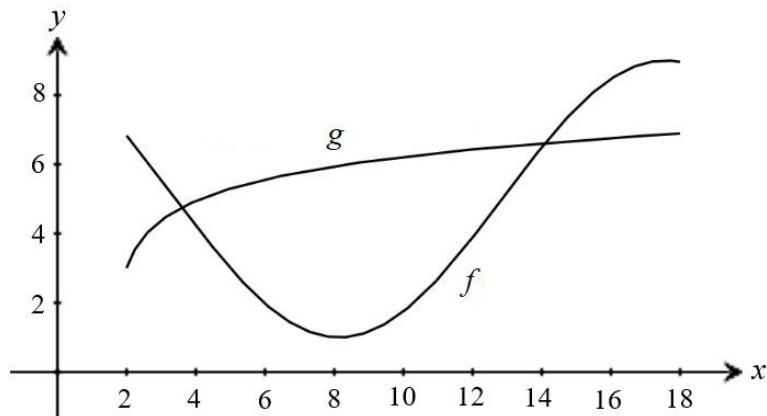
(d) Copy and complete the table below which is the probability distribution for the cost $\$X$. [2]

Cost $\$X$	0.10	0.15	0.25
$P(X = x)$		0.565	

(e) Find $E(X)$. [2]

8. [Maximum mark: 14]

Shown below are the graphs of $f(x) = 4\sin\left(\dfrac{x}{3} + 2\right) + 5$ and $g(x) = \ln(3x - 5) + 3$, for $2 \leq x \leq 18$.

(a) Calculate the area of the region enclosed by the graphs of f and g. [6]

(b) (i) Find $f'(x)$.

    (ii) Find $g'(x)$. [4]

(c) There are two values of x, in the interval $2 \leq x \leq 18$, for which the gradient of f is equal to the gradient of g. Find both these values of x. [4]

Do **not** write solutions on this page.

9.   [Maximum mark: 13]

The heights of players in a basketball league are normally distributed with a mean of 188 cm (correct to three significant figures). It is known that 75% of the players have heights between 179 cm and 194 cm. The probability that a player is shorter than 179 cm is 0.05.

(a) Find the probability that a player is taller than 194 cm.   [2]

(b) (i)   Write down the standardized value, z, for 179 cm.

     (ii) Hence, find the standard deviation of heights.   [4]

To be invited to a special training camp, a player's height must be less than 1.5 standard deviations from the mean.

(c) A player is selected at random. Find the probability that the player is invited to the special training camp.   [3]

30% of the players in the league are women. 75% of the women players are invited to the special training camp.

(d) Given that a player selected at random is invited to the special training camp, find the probability that the selected player is a woman.   [4]